

Using Default Rates to Model the Term Structure of Credit Risk

Jerome S. Fons

As the maturity of a corporate bond increases, its credit spread versus a comparable-maturity Treasury bond may widen or narrow, depending on the bond's credit risk. This bond-pricing model illustrates the relationship between credit spread, estimated default likelihood, and recovery rate. It explains observed patterns in credit spreads, by rating category, as bond maturity varies. Patterns in marginal default rates reflect a typical firm's life cycle. Lower rated (smaller, younger, more heavily leveraged) issuers tend to have wider credit spreads that narrow with maturity. Higher rated (more mature, stable) firms tend to have narrower credit spreads that widen with maturity.

Economists have paid a great deal of attention to the term structure of interest rates but relatively little attention to the term structure of credit risk, defined here as the behavior of credit spreads as maturity varies. Early studies of the term structure of credit risk noted an upward-sloping risk structure for high-grade bonds. That is, the difference, or spread, between the promised yield to maturity of a default-prone bond and the yield to maturity of a default-free bond of equivalent maturity widens as maturity increases. Conversely, researchers found a downward-sloping risk structure for low-grade bonds.

This unusual pattern was explained by a "crisis-at-maturity" model.¹ This model assumes that highly leveraged firms may encounter refinancing problems as their near-term debt matures. Their increased default risk is reflected in higher spreads at shorter maturities. Recent theoretical models based on contingent-claims analysis of debt pricing are much more sophisticated.²

This article introduces a relatively straightforward, risk-neutral model that uses multiperiod corporate bond default rates. This simple approach has the advantage of being highly intuitive and easy to implement. It tells us that, for some levels of default risk, credit spreads indeed decrease with maturity. Although the conclusions of this study are based on the results of Moody's corporate bond default studies, the patterns exhibited can be found in other studies of corporate bond defaults.³

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PATTERNS IN DEFAULT RATES

Moody's default studies summarize the default experience of all corporate, long-term public debt rated by Moody's as of January 1, 1970. More than 473 issuers defaulted between that date and December 31, 1993. By comparing the historical ratings of these defaulting issuers with the ratings for the thousands of public issuers that did not default, we can estimate the default risk associated with each rating category.

The most important concepts the Moody's studies present are marginal and cumulative default rates. To find the marginal default rate, let $m_t^Y(R)$ be the number of issuers rated R (where $R = Aaa, Aa, \dots$) that were originally part of the cohort (or set) of all outstanding issuers with rating R at the start of year Y (where $Y = 1970, 1971, \dots$) and that defaulted in the t th year after Y was formed. Let $n_t^Y(R)$ be the total number of issuers with rating R at the start of year Y that have not defaulted by year t . The marginal default rate, $d_t(R)$, is the average issuer-weighted default rate for R -rated issuers in their t th year. Formally,

$$d_t(R) = \frac{\sum_{Y=1970}^T m_t^Y(R)}{\sum_{Y=1970}^T n_t^Y(R)}, \quad (1)$$

where T equals $(1994 - t)$. The variable T restricts the summations to those cohorts for which t years of history are available—in this example, 1970 through 1993.

For example, $d_5(\text{Baa})$ is the probability that a bond will default in the fifth year after holding the Baa rating. The likelihood that a bond rated R will not default in year t is the marginal survival rate, $1 - d_t(R)$. The probability that a bond rated R will not default by year t is the cumulative survival rate, $S_t(R)$, defined as follows:

$$S_t(R) = \prod_{i=1}^t [1 - d_i(R)]. \quad (2)$$

That is, the cumulative survival rate is the product of the intervening marginal survival rates. The cumulative default rate, $D_t(R)$, is the probability that a bond rated R will default by year t . It is found as follows:

$$D_t(R) = 1 - S_t(R). \quad (3)$$

In other words, the path of marginal default rates through period t completely describes the cumulative survival likelihood, as well as the cumulative default likelihood.

Tables 1 and 2 present marginal and cumulative default rates from 1 to 20 years for Moody's broad rating categories. These estimates are derived from Moody's long-term default studies covering the years 1970 through 1993. The cumulative default rates, in particular, show a clear pattern of increasing risk as rating quality declines over any time horizon.

Table 1. Weighted-Average Marginal Default Rates, by Bond Rating and Investment Horizon, 1970-93

Horizon (years)	Aaa	Aa	A	Baa	Ba	B
1	0.00%	0.02%	0.01%	0.16%	1.79%	8.31%
2	0.00	0.01	0.08	0.35	2.64	7.13
3	0.00	0.05	0.20	0.40	2.66	6.50
4	0.04	0.12	0.18	0.56	2.67	5.52
5	0.09	0.12	0.16	0.52	2.69	4.79
6	0.10	0.11	0.21	0.50	2.19	4.89
7	0.11	0.10	0.24	0.65	1.80	3.59
8	0.12	0.11	0.25	0.68	1.67	3.64
9	0.14	0.13	0.31	0.67	1.67	2.64
10	0.15	0.15	0.35	0.59	1.64	2.57
11	0.17	0.17	0.35	0.64	1.69	1.86
12	0.19	0.20	0.36	0.66	1.74	1.12
13	0.22	0.23	0.35	0.62	1.70	1.22
14	0.25	0.26	0.30	0.73	1.28	1.03
15	0.29	0.00	0.34	0.77	1.24	1.15
16	0.34	0.00	0.35	0.82	1.31	1.30
17	0.21	0.13	0.32	0.85	1.16	1.50
18	0.25	0.16	0.33	0.84	1.06	0.58
19	0.00	0.20	0.40	0.69	1.14	0.00
20	0.00	0.25	0.28	0.71	1.24	0.00

Table 2. Weighted-Average Cumulative Default Rates, 1970-93

Year	Aaa	Aa	A	Baa	Ba	B
1	0.00%	0.02%	0.01%	0.16%	1.79%	8.31%
2	0.00	0.04	0.09	0.51	4.38	14.85
3	0.00	0.08	0.28	0.91	6.92	20.38
4	0.04	0.20	0.46	1.46	9.41	24.78
5	0.12	0.32	0.62	1.97	11.85	28.38
6	0.22	0.43	0.83	2.46	13.78	31.88
7	0.33	0.52	1.06	3.09	15.33	34.32
8	0.45	0.64	1.31	3.75	16.75	36.71
9	0.58	0.76	1.61	4.39	18.14	38.38
10	0.73	0.91	1.96	4.96	19.48	39.96
11	0.90	1.09	2.30	5.56	20.84	41.08
12	1.09	1.29	2.65	6.19	22.22	41.74
13	1.30	1.51	2.99	6.77	23.54	42.45
14	1.55	1.76	3.29	7.44	24.52	43.04
15	1.84	1.76	3.62	8.16	25.46	43.70
16	2.18	1.76	3.95	8.91	26.43	44.43
17	2.38	1.89	4.26	9.69	27.29	45.27
18	2.63	2.05	4.58	10.45	28.06	45.58
19	2.63	2.24	4.96	11.07	28.88	45.58
20	2.63	2.48	5.23	11.70	29.76	45.58

Figures 1 and 2 plot marginal default rates for investment-grade (Aaa, Aa, A, and Baa) and speculative-grade (Ba and B) bonds as the investment horizon grows from 1 to 20 years. These rates represent the probabilities, based on past experience, of an issuer defaulting t years after having an R rating. In general, the trend is for investment-grade marginal default rates to rise as the time horizon lengthens and for speculative-grade marginal default rates to decline. Thus, the year-to-year risk of default does not decrease in the cumulative sense, but it does fall for speculative-grade issuers while rising for investment-grade issuers. This shifting pattern of marginal default rates by

Figure 1. Marginal Default Rates, Investment-Grade Bonds

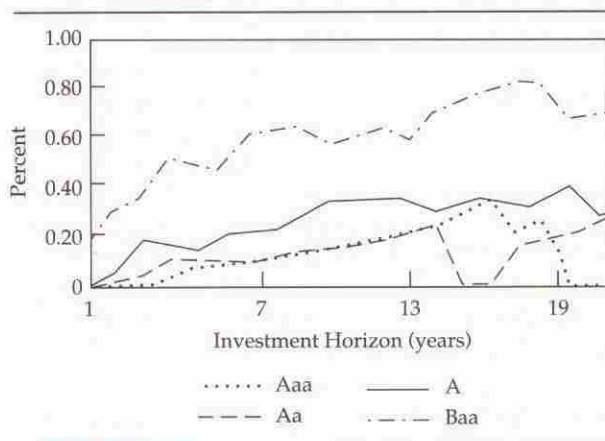
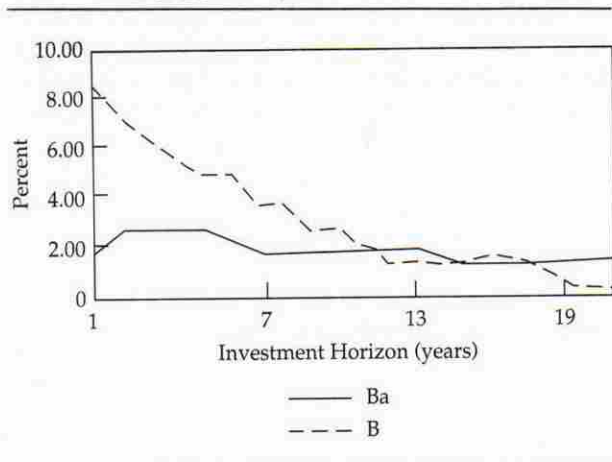


Figure 2. Marginal Default Rates, Speculative-Grade Bonds



rating category, which is also supported by changes in corporate credit quality, suggests an underlying mean reversion in company credit outlooks.

Corporate bond issuers are apt to experience life cycles that cause their ratings to settle at average levels in the long run. Small but growing firms, for example, tend to face a great deal of near-term uncertainty in ability to meet their obligations. The same is generally true of mature firms that alter their capital structures significantly by assuming debt. Either type of firm can be rated as speculative and face substantial near-term risks. Having surmounted such obstacles and survived without a default, an issuer may be upgraded or may pay down the borrowings and withdraw from the public debt markets. In other words, the risk of default ten or more years hence is relatively low once a company survives the first few years.

Investment-grade issuers, in contrast, face very low default risk in the near term. They tend to be large, well-established leaders in their industries with a solid track record of meeting obligations. Their credit outlook for the longer term is somewhat less certain; any number of risks can surface over the course of ten or more years. The industry itself could shrink as new technologies emerge or the political climate changes. Moreover, top-rated firms have only two credit-quality prospects—stable ratings or declining ratings.

Over the long term, then, surviving low-rated issuers tend to rise to the middle ratings, middle-rated firms tend to stay middle-rated, and top-rated firms tend to slip to the middle ratings. This pattern has implications for the term structure of credit risk.

RISK-NEUTRAL BOND PRICING

The default concepts presented above were used in the context of a standard bond pricing formula to develop a yield-spread model for newly issued bonds. A coupon bond paying an annual coupon, C , and maturing N years from now has a yield to maturity, Y , that solves the expression:

$$\text{Price} = \sum_{t=1}^N \frac{C}{(1+Y)^t} + \frac{1}{(1+Y)^N} \quad (4)$$

For simplicity, the price, coupon, and principal terms are expressed as fractions of unity. Equation (4) states that the price of a coupon bond (default prone or default free) is the present value of a stream of cash flows, C , and a terminal principal payment, 1, where the discount rate is Y .⁴

To use the default data to compute a yield spread over a comparable-maturity, default-free debt instrument, we invoked the following four assumptions.

- *Bonds are priced at par.* Typically, a coupon bond is priced at par when first issued. When the price equals 100 percent of par, the coupon rate, C , equals the promised yield to maturity, Y .

- *Investors hold bonds until maturity or default, whichever occurs first.* In the event of default, the bond is sold immediately. This assumption removes the complexity introduced by possible changes in credit rating prior to maturity.

- *Investors are risk-neutral.* That is, they are indifferent between taking a gamble with expected payout X and receiving X with certainty. A risk-neutral individual would be indifferent about participating in a lottery with a ticket price of \$1.00 and an expected payout of \$1.00. Holding the ticket price at \$1.00 and raising the expected payout to, say, \$1.10, however, would lead the utility-maximizing, risk-neutral individual to step up and take a chance.

- *Capital markets are arbitrage free.* In the present context, having arbitrage-free markets means that risk-adjusted expected yields are equal for all securities.

The yield of a default-prone bond is denoted by Y , and the yield of a comparable-maturity, default-free issue by i . If a bond has default risk, its yield must be high enough to compensate investors for the risk; the difference ($Y - i$) is referred to as the credit spread. When the bond is priced at par, the credit spread is ($C - i$).

A bond's risky coupon and principal payments are then weighted by their probability of being paid when promised. Let S_t be the likelihood

that an issuer with a given rating will survive t years from the issuance date without a default. S_t is also the probability that a payment due t years after issuance will be received as promised. Otherwise, the issuer will default on the payment due in year t with probability $S_{t-1}d_t$, which is the probability that the issuer will survive through year $t - 1$ times the probability of a default during year t . In the event of a default, the bondholder sells the bond and expects to recover a fraction of the missed coupon plus principal.

This construct gives us a certainty-equivalent version of Equation 4:

$$\text{Price} = \sum_{t=1}^N \frac{S_t C + S_{t-1} d_t \mu (C + 1)}{(1 + i)^t} + \frac{S_N}{(1 + i)^N}, \quad (5)$$

where $S_0 = 1$. The numerator of the summand is a default-risk-adjusted payment stream based on expected default and survival rates. The probability of receiving a coupon payment t years from now is S_t . A default will occur in year t with probability d_t , given that the bond has survived to year t without a default (the probability of this being S_{t-1}). In the event of default, the investor receives a fraction of $(C + 1)$ representing the coupon plus principal owed. S_N is the likelihood of receiving the final principal payment when due.

Equation (5) is identical to Equation (4) except that the payment stream in Equation (4) is adjusted for default risk. A risk-neutral investor is indifferent between receiving this risk-adjusted payment stream and a certain stream with the same expected value. The appropriate discount rate for this stream is therefore the risk-free interest rate, i .

With the bond price at par (100 percent) and given an array of marginal default rates (which, in turn, imply a set of survival rates), an estimate of the recovery rate, and a default-free yield of appropriate maturity, Equation (5) implicitly determines the risky coupon rate. The difference $(C - i)$ is the credit spread required for the default-prone bond to compensate a buy-and-hold, risk-neutral investor for a set of marginal default rates, d_t , and an expected recovery rate, μ . Such an investor will be indifferent between investing in this bond and investing in an equivalent-maturity, default-free bond yielding i . (Of course, most investors are risk averse and hence would expect to be *more* than compensated for expected default risk, given the uncertainty surrounding point estimates of default likelihood.)

Recovery rates depend on many factors, chief among them is a bond's standing within the firm's

capital structure. Moody's long-term default study estimates recovery rates by bond seniority, based on bond prices one month after default. Table 3 shows that average recovery rates decline uniformly with seniority. For our sample of defaulted bonds, the average recovery rate at the senior unsecured level (the rate associated with most debentures, such as bonds and notes) was 48.38 percent of par, which is the figure used in the simulations presented below.

Table 3. Recovery Rates by Bond Seniority

Seniority	Average Recovery Rate
Senior secured	64.59%
Senior unsecured	48.38
Senior subordinated	39.79
Subordinated	30.00
Junior subordinated	16.33

THE TERM STRUCTURE OF CREDIT RISK

Using the model and historical default and recovery rates, the model spreads were compared with recent spreads in the corporate bond market. Figures 3 and 4 plot risk-neutral spreads from 1 to 20 years for each rating category, as determined from Equation (5). These figures are from the marginal default rates presented in Table 1 and assume a fixed recovery rate of 48.38 percent. To arrive at the appropriate default-free yield for each maturity, we fit a (log-linear) regression model to the U.S. Treasury constant-maturity schedule (as of September 30, 1993) and use the modeled value.

Figure 3. Risk-Neutral Credit Spreads, Investment-Grade Bonds

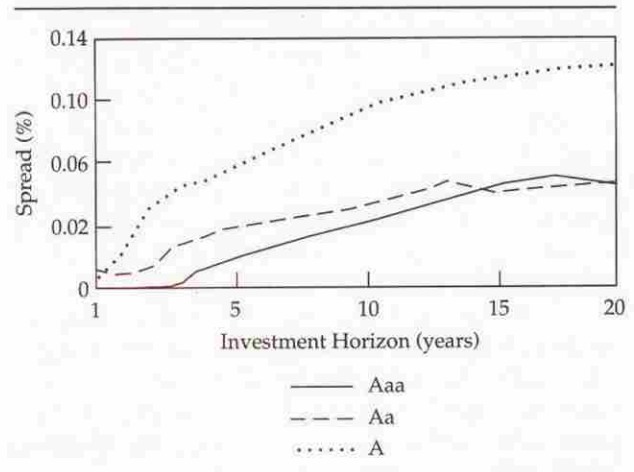
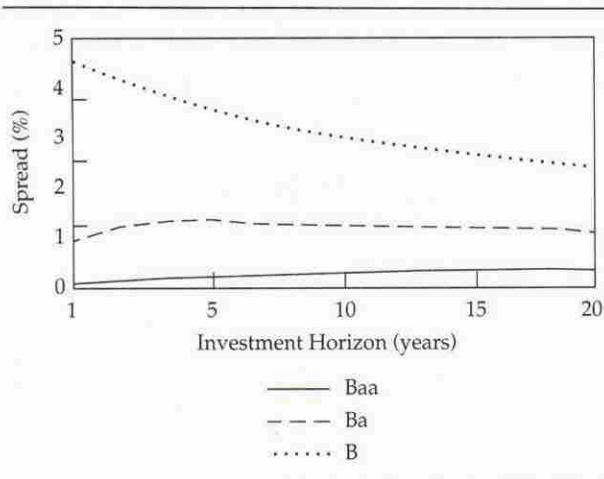


Figure 4. Risk-Neutral Credit Spreads, Speculative-Grade Bonds



Risk-neutral spreads calculated for the investment-grade rating categories exhibit, with minor variations, a steady upward trend as bond maturity increases. To a very small extent, this pattern is attributable to the upward slope of the Treasury yield curve as of September 30, 1993. At the Ba rating level, however, credit spreads rise through the fifth year but then slowly taper off. At the single-B rating, credit spreads fall from Year 1.

Although these patterns roughly match those of the corresponding marginal default rates, it is somewhat surprising to find that spreads against comparable-maturity Treasuries might actually narrow as maturity increases—especially for the lowest rating categories. As the threat of default, even if initially high, recedes, risk-neutral investors require a smaller yield spread to compensate them for expected default loss.

Credit spreads generated using Equation (5) are quite sensitive to the recovery rate estimate. Lowering the estimated recovery rate leads to much wider spreads; higher recovery estimates give much smaller spreads. In contrast, the model is much less sensitive to changes in the level of the risk-free yield.

MARKET BEHAVIOR

Yield spreads calculated using recent market data indicate that our model helps explain observed bond price behavior. We collected yields as of September 30, 1993, for more than 4,000 rated, straight, U.S. corporate bonds. To minimize the effects of call schedules on yields, all bonds callable within one year were eliminated. After deleting major outliers, as well as those bonds with matu-

rities longer than 20 years, we were left with 2,848 bonds—108 bonds rated Aaa, 374 rated Aa, 1,235 rated A, 725 rated Baa, 183 rated Ba, and 223 rated B.

To compute spreads for each of these rating groups, we fit a spline regression to the U.S. Treasury yield curve as of September 30, 1993. The six panels in Figure 5 show the resulting market credit spreads for each rating category. These scatter plots include a line fitted using standard linear regression methods.

No clear trend emerges in the plot of Aaa spreads against comparable-maturity Treasury bonds as the term varies. Bonds rated Aa and A, however, exhibit a significant, positive relation between spread and maturity. Bonds rated Baa also show a positive relationship, although not as strong as that for bonds rated A.

Consistent with the modeled behavior, credit spreads for bonds rated Ba decline only slightly as maturity increases. As anticipated, we found a significant negative spread-maturity relationship for B-rated bonds. Furthermore, the lower bounds of the Aa and A spreads exhibit the convexity seen in their corresponding theoretical spread models.

The regression results presented in Table 4 support these observations. In part because of the noise contained in the price data, the overall fit of the regressions is quite low. In two cases—bonds rated Aaa and bonds rated Ba—the slope coefficients of lines fitted through spreads as a function of maturity are not significantly different from zero. All other slope terms are significantly different from zero at the 5 percent level of confidence. The A-rated bonds have the steepest positive slope, whereas the spreads for B-rated bonds are negatively related to maturity. All intercept terms appear to be significant at the 5 percent level, and they rise as credit quality falls.

For each rating category, the fitted line lies above its theoretical counterpart. The difference is particularly apparent for the investment-grade categories. For these bonds, the 20-year theoretical credit spread ranges between 0.06 percent and 0.31 percent, and the actual 20-year maturity market spreads (as of September 1993) averaged 0.74 percent for bonds rated Aaa, 1.14 percent for bonds rated Aa, 1.27 percent for bonds rated A, and 1.85 percent for bonds rated Baa.

Theoretical risk-neutral spreads for speculative-grade bonds more closely fit measured spreads. The model spreads for Ba-rated bonds peak at 1.35 percent in Year 5, whereas market spreads for these bonds averaged 3.37 percent.

Figure 5. Market Spread by Maturity, Bonds of Various Ratings

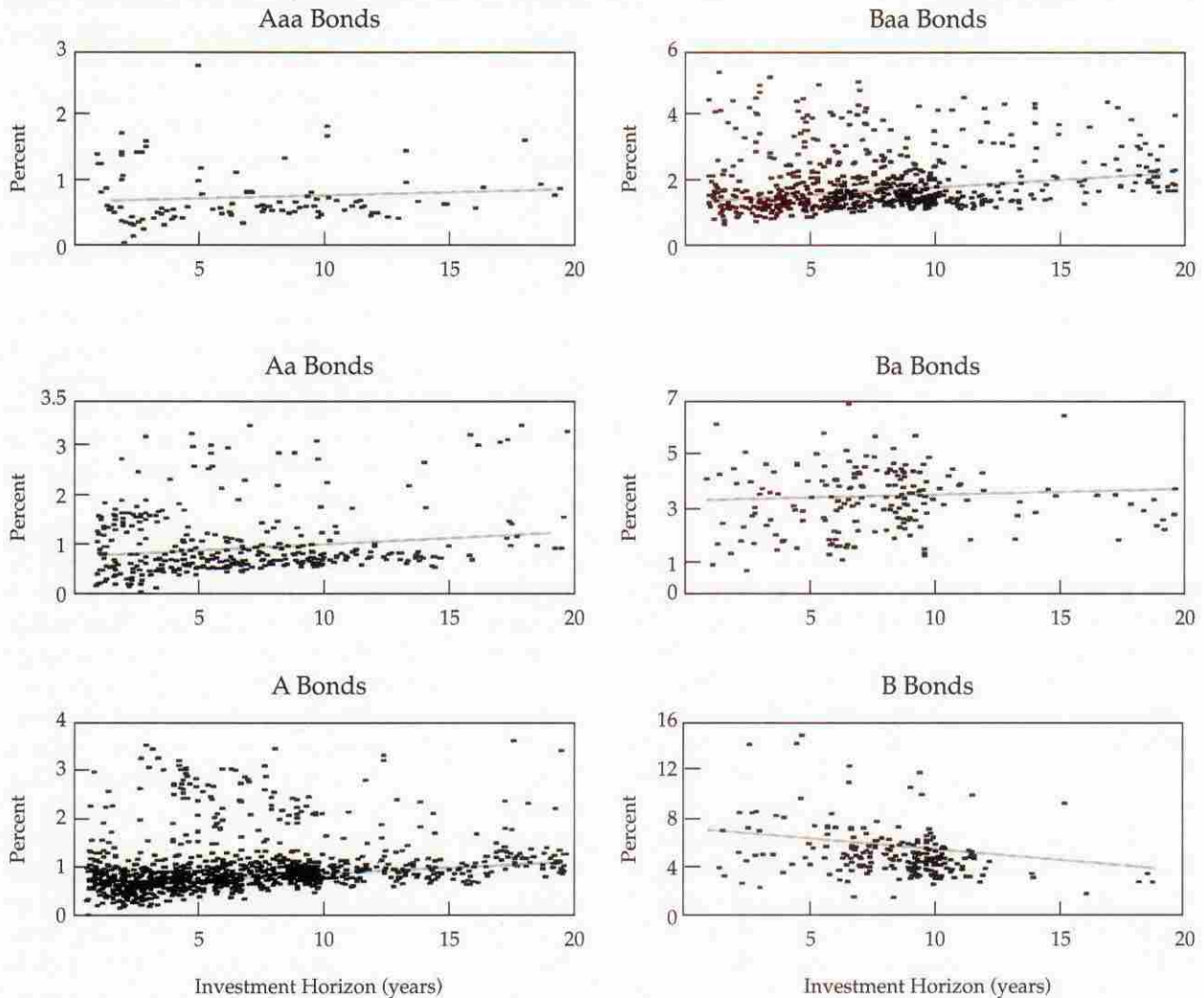


Table 4. Regression Results

Bond Rating	Slope Coefficient	Intercept	Adjusted R ²
Aaa	-0.0003 (-0.04)	0.742 (10.04)	-0.94%
Aa	0.015 (2.13)	0.837 (14.70)	0.94
A	0.0174 (4.89)	0.916 (31.63)	1.82
Baa	0.0212 (2.96)	1.425 (23.02)	1.06
Ba	-0.002 (-0.09)	3.372 (18.36)	-0.55
B	-0.208 (-4.71)	7.254 (19.45)	8.71

Note: Numbers in parentheses are *t*-statistics.

Based on historical default rates, risk-neutral spreads for B bonds should range between 4.64 percent (at 1-year maturity) and 2.37 percent (for 20-year maturity). Market spreads for B bonds averaged 7.05 percent at 1 year and 3.09 percent at 20 years.

The discrepancies between the model estimates and market credit spreads have several explanations. One is that, by virtue of its size and activity level, the liquidity of the market for U.S. Treasury securities far exceeds that of any other bond market. Investors in less liquid issues may require a premium as compensation for liquidity risk.

Second, we assumed that bond investors are

risk-neutral. Although individuals may exhibit risk-neutral behavior with small portions of their wealth (such as lotteries), they are likely to be risk-averse when large sums are concerned. Default-rate (as well as recovery-rate) estimates contain higher order risk factors.

Third, we assumed that investors could not sell their bonds except in the case of default. Clearly, individuals are free to trade bonds, and they may suffer paper losses if the creditworthiness of their bonds declines after the purchase date. Investors in high-rated bonds, in particular, may want compensation for this risk.

Fourth, interest earned on U.S. Treasury securities is exempt from state and local income taxes. As a result, individual investors, in particular, are often willing to accept a lower yield on these securities.

Finally, although we made an effort to minimize complications resulting from the call provisions found in many corporate bonds, residual effects remain. When a firm has the right to call its bonds, investors will demand compensation in the form of a higher promised yield.

As noted earlier, risk-neutral credit spreads are quite sensitive to the recovery rate estimate, but the derived risk-neutral spreads should establish a lower bound on actual market spreads. Although introducing risk aversion will lead to wider theoretical spreads, other risks play a role in the pricing of corporate bonds. These risks could, under certain circumstances, dominate default risk.

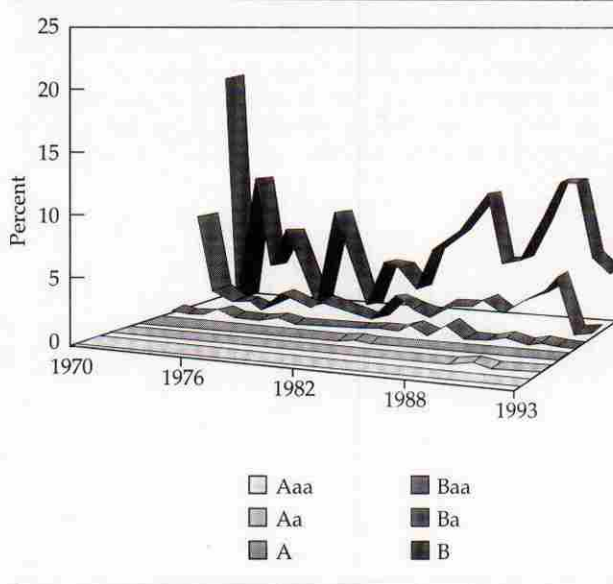
STABILITY OF DEFAULT-RATE ESTIMATES

The default rates used in the pricing model represent point estimates. History shows that default rates are not particularly stable, especially at low rating levels. By taking the number of issuers with a given rating that default in a year and dividing by the total number of issues with that rating at the beginning of the year, we can form a time series of one-year default rates by rating category.

Figure 6 illustrates the increase in default rate volatility at the lower rating categories after 1970. The figure also suggests at least a mild correlation

between default rates and the overall business cycle. The recessions of 1974, 1982, and 1990, as well as the slowdown of 1985, were each accompanied by surges in speculative-grade default rates. (The large spike in 1970 reflects the default of Penn Central Railroad and its affiliates.) Broader trends in corporate leverage are a factor as well, particularly in the last half of the 1980s.

Figure 6. One-Year Default Rates by Rating Category, 1970-93



The increased volatility of default rates at lower rating categories, as well as their cyclical nature, introduce uncertainty in pricing. Risk-averse investors may require added compensation for this uncertainty. There is also (although the evidence is not presented here) considerable volatility across recovery rates (measured as the bond's trading price one month after default). This phenomenon also might induce risk-averse investors to seek compensation above actuarially neutral yields.

Left for future research is the question of how the spread-versus-maturity relationship might evolve through time and its role as an indicator of the timing and phases of the credit cycle.⁵

FOOTNOTES

1. See R. E. Johnson, "The Term Structure of Corporate Bond Yields as a Function of Risk of Default," *Journal of Finance*, vol. 22, no. 2 (1967):313-45; J. B. Silvers, "An Alternative to the Yield Spread as a Measure of Risk," *Journal of Finance*, vol. 28, no. 4 (1973):933-55; and J. C. Van Horne, *Financial Market Rates and Flows* (Englewood Cliffs, N.J.: Prentice-Hall, 1978):164-73.
2. See L. T. Nielsen, J. Saa-Requejo, and P. Santa-Clara, "Default Risk and Interest Rate Risk: The Term Structure of Default Spreads," INSEAD Working Paper (May 1993); I. J. Kim, K. Ramaswamy, and S. Sundaresan, "Does Default Risk in Coupons Affect the Valuation of Corporate Bonds? A Contingent Claims Model," *Financial Management*, vol. 22, no. 3 (Autumn 1993):117-31; A. Ginzburg, K. Maloney, and R. Willner, "Risk Rating Migration and the Valuation of Floating Rate Debt," Working Paper (December 1993); E. P. Jones, S. P. Mason, and E. Rosenfeld, "Contingent Claims Analysis of Corporate Capital Structures: An Empirical Investigation," *Journal of Finance*, vol. 39, no. 3 (1984):611-25; and R. Litterman and T. Iben, "Corporate Bond Valuation and the Term Structure of Credit Spreads," Goldman Sachs Financial Strategies Group (1988).
3. As a matter of policy, Moody's Investors Service does not make buy or sell recommendations. The model presented here is just one of many possible ways to factor credit risk into the price of a bond.
4. We assume annual coupon payments to simplify the mathematics. The analysis extends easily to semiannual coupon payments.
5. The author thanks Douglas Lucas, Roger Stein, and Pat Corcoran for their comments and Lea Carty for his assistance and comments.

