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The Incorporation of Joint-Default Analysis into Moody's Corporate, Financial and Government Rating Methodologies

For many obligations, the risk of default depends on the performance of both the primary obligor and another entity. This Special Comment describes Moody's approach to the assignment of ratings to obligations linked to the credit profiles of two obligors.

Specifically, we consider three paradigmatic situations:

- Guarantees – in which the secondary party unconditionally guarantees or provides an irrevocable letter of credit in support of the primary obligor;
- Partial support – in which the secondary party (e.g., the issuer's parent) provides a limited guaranty or implicit (but uncertain) support; and
- Interference – in which a secondary party in distress, such as a sovereign, might take actions that cause the primary obligor to default, such as a deposit freeze or the imposition of foreign currency transfer restrictions.

Historically, Moody's has typically rated:[1]

- Fully guaranteed transactions at the higher of the two obligors' ratings, using a so-called "credit substitution" approach;
- Partially supported transactions at a level between the ratings of the primary and secondary obligors; and
- Transactions subject to interference at the lower of the two obligors' ratings, according to a so-called "weakest link" approach, or, in the case of foreign currency transfer risk, at the sovereign "ceiling."

While easy to understand, communicate and implement, these approaches may lead to ratings that are too conservative if the correlations are imperfect among the events of default for the two obligors. Moreover, these approaches make it difficult to ascertain explicitly the degree of partial support – or the likelihood of interference – embodied in a rating.

1. *Please see the Bibliography at the end of this document for a list of related publications.*

Instead, this Special Comment describes an analytical approach which begins with a systematic breakdown of relevant risks into the following fundamental components:

- obligor ratings that summarize the "standalone" risk of failure (or need for rescue) by the primary and secondary obligors;[2]
- the probability that the secondary obligor would provide support to, or interfere with, the primary obligor; and
- the standalone default correlation between the primary and secondary obligors.

We illustrate below how these risk components can be combined to infer the joint probability of default – and thereby infer an analytically consistent and transparent rating to be assigned to the jointly supported (or interfered with) obligation. It is important to note that, under certain circumstances, a supported issuer's rating could exceed that of a supporting entity and ratings of obligors that are subject to interference risk might exceed the rating of a potentially interfering entity.

Currently, Moody's employs variations on what we designate here as "joint-default analysis" within several rating areas, including structured finance, letter-of-credit-backed bonds and certain obligations subject to a sovereign ceiling.[3] This document provides a framework for the extension of this approach to other areas on a sector-by-sector basis. As we prepare to incorporate the new methodology we will provide prior methodological guidance specifically tailored to representative issuers in that sector.

Moody's anticipates applying the methodology first to government-related issuers, followed by regional and local governments and then to the banking sector.

JOINT-DEFAULT ANALYSIS AND EXAMPLES

Rating analysts frequently confront the problem of rating a jointly supported obligation, where two obligors promise unconditionally to see that debt service is paid on a timely basis. Since a default on such an obligation will occur only if both parties default, the rating on a jointly supported obligation should be at least as high as the rating of the higher-rated obligor. In order to determine in a consistent fashion whether the supported rating should be higher than the stronger obligor's rating, however, one must examine the relationship between the two obligors.

Intuitively, the amount of "lift" that the jointly supported obligation can achieve relative to the stronger obligor's rating will depend on two factors: the rating of the weaker obligor and the correlation between default events for the two obligors. As a general rule, the lift should be higher, the higher the rating on the weaker entity and the lower the correlation. The following discussion provides a framework for quantifying these considerations. We discuss a related topic, ratings in the presence of "partial" support, after addressing situations involving certain support or formal guarantees. Lastly, we extend this framework to scenarios in which a weaker entity instead interferes with a stronger entity.

CONDITIONAL DEFAULT PROBABILITIES

The probability that two parties will jointly default depends on a) the probability that one of them defaults, and b) the probability that the second will default, given that the first has already defaulted. Expressed algebraically, one can write this for events **A** and **B** as:[4]

$$P(\mathbf{A} \text{ and } \mathbf{B}) = P(\mathbf{A} \mid \mathbf{B}) \times P(\mathbf{B}) \qquad (1)$$

Or equivalently,

$$P(\mathbf{A} \text{ and } \mathbf{B}) = P(\mathbf{B} \mid \mathbf{A}) \times P(\mathbf{A}) \qquad (2)$$

We define **A** as the event "obligor *A* defaults on its obligations" and **B** as the event "obligor *B* defaults on its obligations." Likewise, "**A** and **B**" is the joint-default event "obligors *A* and *B* both default on their obligations."[5] The operator P(•) represents the probability that event "•" will occur and P(• | *) is defined as the conditional probability of event "•" occurring, given that event "*" has occurred.

2. *Although such ratings may not always be published, we anticipate that a supporting party will generally have a published rating.*
3. *See, for example, "Moody's Approach to Jointly Supported Obligations," Moody's Special Report, November 1997 and "Revised Country Ceiling Policy," Moody's Rating Methodology, June 2001.*
4. *Statisticians will recognize these equations as axioms of probability theory that underlie Bayes' Theorem.*
5. *The implication here is that the default events occur simultaneously, but we require only that the timing be such that a holder of the supported obligation suffers credit loss within a specified horizon.*

Moody's ratings can be used to infer directly the probability that a particular issuer will default (P(**A**) and P(**B**)).[6] But in order to estimate the conditional default probabilities P(**A** | **B**) and P(**B** | **A**), one must take into account the relationship between the drivers of default for both obligors. Each of these four probabilities – P(**A**), P(**B**), P(**A** | **B**) and P(**B** | **A**) – are intended to represent unsupported risk measures. That is, they represent the likelihood of an obligor default in the absence of any joint support or interference. We present in the Applications and Examples section below a framework for modeling both support and interference.

Although in theory, one can tackle this problem directly by estimating either one of the conditional default probabilities described in equations (1) and (2), it may be more intuitive to focus on the product of the conditional probability of default for the lower-rated, or supported, firm and the unconditional probability of default for the higher-rated, or supporting, firm. Using **L** to denote the event "lower-rated obligor *L* defaults on its obligations" and **H** to denote "higher-rated obligor *H* defaults on its obligations," we can rewrite equation (1) as:

$$P(\mathbf{L} \text{ and } \mathbf{H}) = P(\mathbf{L} \mid \mathbf{H}) \times P(\mathbf{H}) \tag{3}$$

It is not difficult to imagine situations where the conditional probability P(**L** | **H**) might be at its theoretical maximum (i.e., 1) or at its minimum (i.e., P(**L**)).[7] Let us consider these extreme outcomes in turn by way of example.

- P(**L** | **H**) = 1. Suppose that the financial health of an issuer is crucially linked to the operations of another, higher-rated entity. For example, the default risk of a distributor in a competitive distribution market dominated by a single supplier may be highly dependent on the financial health of that supplier. In other words, the conditional probability of the distributor's default given a default by the higher-rated supplier, P(**L** | **H**), is equal to one. In this case, events **L** and **H** are maximally correlated.[8] Under such a scenario, the joint-default probability P(**L** and **H**) in equation (3) above is simply P(**H**). That is, the rating applied to such jointly supported obligations would equal the supplier's rating, without any ratings lift, regardless of issuer **L**'s standalone rating.
- P(**L** | **H**) = P(**L**). Suppose a highly rated European bank provides a letter of credit to a lower-rated agribusiness in the US. While there may be circumstances in which the agribusiness might face financial difficulties on its own, its intrinsic operational health is generally unrelated to the circumstances that might lead the European bank to default on its obligations. Under this scenario, the conditional probability of a default by the agribusiness, given a default by the bank – i.e., P(**L** | **H**) – is simply the standalone default risk P(**L**) of the agribusiness. That is, events **L** and **H** are uncorrelated and independent of one another. In this case, their joint-default probability is the product of their standalone default probabilities, P(**L**)*P(**H**). The jointly supported obligation rating implied by such a relationship is generally higher than the rating of the supporting entity **H**.

In practice, the conditional default risk of the lower-rated entity, given a default by the stronger entity, will vary somewhere between these two extremes, maximum correlation (i.e., where P(**L** | **H**) = 1) and independence, (i.e., where P(**L** | **H**) = P(**L**))

INTERMEDIATE LEVELS OF CORRELATION

We propose here a simple tool for modeling intermediate cases of default risk linkage. Let us denote the variable W as a correlation weighting factor, where W = 1 corresponds to a maximum theoretical correlation between the default of the lower-rated entity and that of the higher-rated entity; and W = 0 corresponds to a complete independence (i.e., zero correlation) between default events. Fractional values of W indicate intermediate levels of correlation between the two default events.

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6. *Moody's ratings are defined as ordinal (or relative) measures of default risk and not in terms of cardinal (or absolute) default rates. However, as long as ratings can provide a constant measure of relative default risk, with actual default probabilities rising and falling proportionately by rating category over a credit cycle, the methods proposed here will produce logically consistent measures of jointly supported ratings.*
7. *Technically, the conditional default probability P(**L** | **H**) could be as low as zero, a situation which would occur if the default correlation between the two obligors was at its theoretically maximum negative value. However, throughout our discussion, we follow the standard practice of ignoring the highly unlikely possibility that the default experience of the two obligors will be negatively correlated.*
8. *This use of the term "correlation" applies to default events that follow a binomial distribution and should not be confused with potential correlation in rating transitions (or default intensities). When the default profiles of two obligors are maximally correlated, P(**L** | **H**) = 1 and P(**H** | **L**) = P(**H**)/P(**L**). That is, the weaker entity always defaults when the stronger entity defaults, and the stronger entity will only default if the weaker entity also defaults. This leads to the result P(**H** | **L**) = P(**H**)/P(**L**). **Note that maximum correlation will be less than 1 in cases where obligors have different ratings.***

Using the correlation weighting concept, we can express the joint-default probability between obligors L and H as:

$$P(\mathbf{L} \text{ and } \mathbf{H}) = W^* P(\mathbf{L} \text{ and } \mathbf{H} \mid W=1) + (1-W)^* P(\mathbf{L} \text{ and } \mathbf{H} \mid W=0) \tag{4}$$

Or more compactly,

$$P(\mathbf{L} \text{ and } \mathbf{H}) = W^*P(\mathbf{H}) + (1 - W)^*P(\mathbf{L})^* P(\mathbf{H}) \tag{5}$$

In other words, once we have determined standalone ratings for the two obligors, the task of assigning a rating to a jointly supported obligation may be reduced to the assignment of a correlation weight.[9]

APPLICATIONS AND EXAMPLES

1) Full Support and Guaranteed Obligations

A natural application of the approach described above is to the assignment of a rating on an obligation fully supported by two obligors. That is, both obligors are jointly and severally liable for payment. A payment default on the obligation occurs only when both parties default. Guarantees fall under this scenario.

Assume, for example, that an obligation is fully supported by two obligors, one (issuer L) that is rated Baa2 and the other (issuer H) that is rated A1. As noted above, Moody's ratings can be used to infer a probability of default over a given investment horizon. One might, for example, use the following look-up table to convert ratings into default probabilities (and vice versa):[10]

Rating	Default Probability
Aaa	0.00%
Aa1	0.02%
Aa2	0.05%
Aa3	0.10%
A1	0.19%
A2	0.35%
A3	0.54%
Baa1	0.83%
Baa2	1.20%
Baa3	2.38%
Ba1	4.20%
Ba2	6.80%
Ba3	9.79%
B1	13.85%
B2	18.13%
B3	24.04%
Caa1	32.86%
Caa2	43.88%
Caa3	66.24%

*9. While this derivation focused on P(**L** | **H**), it could also be approached through a focus on P(**H** | **L**). (See footnote 8.) An alternative methodology is described in a paper published by Douglas Lucas, "Default Correlation and Credit Analysis," The Journal of Fixed Income, Vol. 4, No. 4, March 1995.*

10. A default probability table is necessary to implement the methodology; however, only relative default probabilities – not their absolute values – matter to the analysis. For example, the critical feature of this table is that issuers rated A1 are assumed to default at nearly twice the rate of those rated Aa3, and not that the respective expected default rates are 0.19% and 0.10%, per se. This particular table represents four-year idealized default rates as shown in "Moody's Approach to Rating Synthetic CDOs," Moody's Structured Finance Rating Methodology, July 29, 2003.

Based on this table, P(**L**) = 1.20% and P(**H**) = 0.19%. Let us define "medium correlation" as implying a weight W of 50% – i.e., equal weight on the maximum correlation and complete independence scenarios. Substituting these values into equation (5) gives a joint-default probability P(**L** and **H**) of 0.10%. Using the above look-up table, we see that this corresponds to the Aa3 rating, and therefore implies a one-notch lift, as compared to a credit-substitution approach.

Raising the correlation weight to 75% – i.e., relatively high correlation – yields an implied joint-default probability of 0.14%, geometrically closer to the A1 rating in the look-up table above. On the other hand, setting the correlation weight to 25%, corresponding to a low correlation scenario, yields a joint-default probability of 0.05%, equivalent to the Aa2 rating.

2) Partial Parent or Government Support

In many cases, an obligation benefits from external support, but that support falls short of an iron-clad guarantee. Examples include bonds issued by a weak subsidiary of a relatively strong parent firm, or bonds issued by an issuer with partial government ownership. In the latter case, the government's incentive to bail the issuer out, should it run into difficulties, may be a function of the share of government ownership or of the importance of that issuer to the national economy.

Prior Moody's publications on the role of external support detail the incentives parent firms and governments face in their decision to support a troubled entity. Ownership share, legal basis for support and strategic importance all factor into the decision.[11]

It is helpful to think of the two extreme situations in which an investor faces losses. The first is where the issuer of the obligation defaults and there is no external support. The probability of this event occurring is simply P(**L**), the probability that issuer *L* will default on its own. The second is where there is full support, but both the issuer and the support provider default on their obligations. As above, this is given by P(**L** and **H**). The degree of support can also be thought of as a probability and can therefore vary between 0 and 1. We model the risk to the investor as a shifting probability between the two risk outcomes P(**L**) and P(**L** and **H**):

$$P(\mathbf{L} \text{ and } \mathbf{H} \mid S) = (1-S)*P(\mathbf{L})+S*P(\mathbf{L} \text{ and } \mathbf{H}) \quad (6)$$

Here, the weighting parameter S represents the likelihood of support. Full support (i.e., S = 1) leads to the joint-default outcome and no support (i.e., S = 0) yields the standalone default risk of the obligor, P(**L**).

Consider the situation outlined in the example above, with the correlation weight W set at 50%. Recall that the resulting joint-default probability was 0.10%, equivalent to the Aa3 rating. If we assume the likelihood of parent support is also at the 50% level (suggesting that the likelihood of the higher-rated entity extending support is equal to a coin toss), then the resulting implied default probability from equation (6) is 0.65%. Corresponding roughly to the A3 rating, this figure lies halfway between the standalone default risk of the weaker entity (1.20%) and the joint-default probability (0.10%). Raising support to 90% yields an implied default probability of 0.21%, for a rating of A1. Lowering support to 30% yields an implied default probability of 0.87%, for a rating of Baa1.

3) Interference and Piercing the Sovereign Ceiling

In certain situations, a default by a secondary party leads to actions (or "interference") which cause the primary obligor to default. For example, in emerging market economies, we sometimes face a situation where an entity – on a standalone basis – has a stronger credit profile than the government of the country in which it is domiciled. But because government defaults are sometimes accompanied by a freeze on foreign currency convertibility, the issuer is rarely rated higher than the government. In other cases, a weak parent may have an incentive to de-capitalize a subsidiary in order to save itself.

Under such circumstances an investor will suffer losses if either: 1) the primary obligor defaults (irrespective of the government's or parent's situation); or 2) the government (or parent) defaults and interferes with the primary obligor's ability to operate, leading to the primary obligor's default.

In order to model this "or" situation, we define **L** and **H** as above. Let us also define **I** as the event "obligor *L* interferes with obligor *H*." The problem can be summarized as:

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11. Please see "Rating Non-Guaranteed Subsidiaries: Credit Considerations in Assigning Subsidiary Ratings in the Absence of Legally Binding Parent Support," Moody's Rating Methodology, December 2003; "State-Owned Quasi -Corporate Entities (SQEs): Moody's Rating Approach," Moody's Rating Methodology, May 2003; and "How Moody's Evaluates Support Mechanisms Provided By Parents, Affiliates Or Other Related Entities," Moody's Rating Methodology, December 1999.

$$P(\mathbf{L} \text{ or } \mathbf{H} \mid \mathbf{I}) = P(\mathbf{H}) + P(\mathbf{L} \text{ and } \mathbf{I}) - P(\mathbf{L} \text{ and } \mathbf{H} \text{ and } \mathbf{I}) \quad (7)$$

If we assume that interference **I** is independent of default events **L** and **H** then we have:

$$P(\mathbf{L} \text{ or } \mathbf{H} \mid \mathbf{I}) = P(\mathbf{H}) + P(\mathbf{L})^*P(\mathbf{I}) - P(\mathbf{L} \text{ and } \mathbf{H}^*P(\mathbf{I})) \quad (8)$$

The last term is subtracted because this is a dual-risk scenario and we want to avoid double counting default risk.

In recent years, we have witnessed situations in which a government default was not accompanied by a currency freeze, as well as situations in which a currency freeze was imposed but some issuers were exempt and therefore allowed to continue servicing their obligations in foreign currency. This represents the special case of partial interference. Let us represent the event "imposition of a payment moratorium" by **M** and the event "issuer *H* is caught up in the moratorium" by **C**. If the default events **L** and **H** are both independent of **M** and **C**, then we can calculate the risk that bondholders of issuer *H* will suffer credit losses as:

$$P(\mathbf{L} \text{ or } \mathbf{H} \mid \mathbf{M}, \mathbf{C}) = P(\mathbf{H}) + P(\mathbf{L})^*P(\mathbf{M})^*P(\mathbf{C}) - P(\mathbf{L} \text{ and } \mathbf{H}^*P(\mathbf{M})^*P(\mathbf{C})) \quad (9)$$

Using the default table from the examples above, if issuer *H* is rated A3 on a standalone basis, the government *L* is rated Ba3 and we assume medium correlation (i.e., W = 50%), then with a 50% probability that a moratorium would be imposed (in the event of a government default) and a 50% probability that issuer *H* will be caught up in such a moratorium, the implied rating would equal Baa3. Raising the probability that issuer *H* will be caught up in the moratorium to 90% lowers the implied rating to Ba1, still two notches higher than the government's rating of Ba3.

When Moody's first introduced this methodological change in June 2001, it was thought that only issuers with significant foreign exchange earnings would be eligible to pierce the country ceiling. However, as more issuers were rated using the refined methodology, it soon became clear that – depending upon the specific government bond rating and the issuer's local currency rating, as well as the probability that a moratorium would be used as a policy tool – even issuers with no foreign exchange earnings were potentially eligible to pierce the ceiling. This is especially true for countries where Moody's sees a low risk of a foreign currency moratorium being used.[12]

12. *Please refer to Moody's Rating Methodology "Revised Country Ceiling Policy," June 2001.*

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